Dimensional Synthesis of One-Jointed Multi-fingered Hands

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Abstract. Wristed, multi-fingered hands can be designed for specific tasks, leading to an optimized performance and simplicity. In this work we present the design of the simplest family of multi-fingered hands, with one revolute joint at the wrist and a set of fingers attached to the palm with a single revolute joint each. It is shown that hands with two to five fingers can be designed for meaningful tasks, and that two arbitrary positions can be defined at most for these hands, yielding a good tool for pick-and-place, or grasp-and-release, tasks. For each of those possible designs, dimensional synthesis is performed and an algebraic solution is derived. It is proved that two solutions exist for the general case of this family of hands. Coupled actuation for the grasp-and-release task can be easily implemented for these hands, to create an underactuated design able to be driven with a single actuator. Some examples are presented.

Key words: Robotic hands, Kinematic synthesis

1 Introduction

Kinematic chains with a tree topology consist of several common joints that branch to a number of serial chains, each of them corresponding to a different end-effector. A typical example of a kinematic chain with a tree topology is a wristed, multi-fingered hand.

Compared to other topologies, the tree topologies have not been so widely studied. Kinematic analysis for applications in modular robots and robotic hands can be found in [10], [11], and [1], and dynamic analysis is found in [3] and [2]. Structural synthesis for multiple fingers with no wrist, considering grasping and manipulation requirements, are found in [4]. The first reference to kinematic design of tree topologies is found in [7].

The kinematic synthesis of these topologies presents particular challenges that are different of those that appear in single serial chains or in closed-loop systems. In particular, the kinematic synthesis of multi-fingered hands has been explored also in [9] and more extensively in [8].

When dealing with exact kinematic synthesis, one of the first steps is to define the task in order to size the workspace of the system. In particular, we considere revolute joints without joint limits. In the case of tree topologies, consider a task having the same number of positions for each of the multiple end-effectors; this means that we are dealing with a coordinated action of all those fingertips, denoted as a *simultaneous task*. It has been proved [5] that not all tasks are possible for all fingers, as the relative motion between fingers needs to be considered.

In [5], a chart of solvable multi-fingered hands with identical fingers was presented. Here we focus on the simplest case of that chart, a family of hands with one revolute joint at the wrist and a series of fingers, joined to a single palm by one revolute joint each. For this family of hands, it is possible to obtain a closed, algebraic solution for the design equations. This provides a fast calculation method, as well as information on the number of solutions and the effect of pre-defined constraints. The method is demonstrated with an example.

2 Multi-fingered Hands

A tree topology for a kinematic chain has a set of common joints splitting on several chains, possibly in several stages, and ending in multiple end-effectors [6]. The tree topology is represented as a rooted tree graph; for this we follow the approach of Tsai [13], the root vertex being fixed with respect to a reference system. In tree topologies, a vertex can be connected to several edges defining several branches.

Wristed, multi-fingered hands are kinematic chains with a tree or hybrid topology. For our synthesis formulation, the internal loops in the hand structure are substituted using a reduction process [8], so that the hand has a tree topology with links that are ternary or above.

Tree topologies are denoted as $W-(B_1,B_2,\ldots,B_b)$, where W are the common joints and the dash indicates a branching or splitting stage, with the branches contained in the parenthesis, each branch B_i characterized by its type and number of joints. In the case of using just revolute joints, the joint type is dropped and only the number of joints is indicated. Figure 1 shows the compacted graph for the $R-(R,R,R,R,R)$, or $1-(1,1,1,1,1)$ chain. This is one of the member of the one-jointed hand family, in particular the five-fingered hand. The root vertex is indicated with a double circle.

3 Dimensional Synthesis of Multi-fingered Hands

Given a desired hand topology with b branches and joint axes S_i, and a simultaneous task for each fingertip, characterized by a set of m_p finite positions $\hat{P}^b_{1k}$ and m_v velocities $\dot{\mathsf{P}}^b_k$, kinematic synthesis is applied by equating the forward kinematics equations of each branch to the relative displacement of the corresponding fingertip. Similarly, velocities can be defined for some of those task positions,

$$\mathbf{F}(\mathsf{S},\Delta\theta,\dot{\theta}) = \begin{cases} \hat{P}^c_{1k} - \prod_{i=1}^{n_c} e^{\frac{\Delta\theta^k_{i,c}}{2}\mathsf{S}_{i,c}}, & k = 2,\dots,m_p \\ & c = 1,\dots,b \\ \dot{\mathsf{P}}^c_k - \sum_{i=1}^{n_c} \dot{\theta}^k_{i,c}\mathsf{S}^k_{i,c}, & k = 1,\dots,m_v \\ & c = 1,\dots,b \end{cases} \tag{1}$$

In most of the cases, this set of equations is solved using numerical methods to obtain a kinematic design.

4 Single-jointed, Single-branching Hands

The solvability of this family of hands was studied in [5]. In Table 1, all possible one-jointed trees consisting of revolute joints and able to perform a meaningful task are presented. The first row in the table contains the serial $R-R$ robot, which is known [12] to be solvable for $m = 3$ positions, yielding two real solutions. The last row presents the $1-(1,1,1,1,1)$ hand, with five fingers. Notice that any general hand of these characteristics with more than five fingers will not be solvable for a useful task, as it will not be able to reach an initial and final positions. For all the cases in the middle, from two to four fingers, the hand will be able to reach an initial and a final position, as their solvability calculations show two plus one incompletely specified position.

5 Dimensional Synthesis for the $1-(1,1,1,1,1)$ Hand

The $1-(1,1,1,1,1)$ hand consists of a single revolute joint at the wrist and a palm spanning five fingers, each of them being a fingertip link connected to the palm by a single revolute joint. Figure 1 shows the reduced tree graph for the hand and the kinematic sketch with the notation used for the axes.

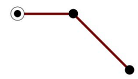
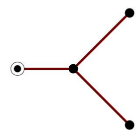
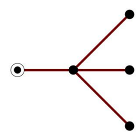
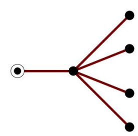
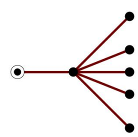
The six joints have Plucker coordinates $\mathsf{S}_i = \mathbf{s}_i + \varepsilon\mathbf{s}_{i0}$, for $i = 0,\dots,5$. The rotation angle about each joint is θ_i.

As stated in Table 1, this kinematic system can be solved for exact dimensional synthesis for $m = 2$ positions. In what follows, the algebraic solution is derived.

5.1 Design Equations

Let $\hat{P}_{ij}$ be the i^{th} displacement assigned to finger j. For this robot, $i = 1,2$ and $j = 1,\dots,5$. Construct the relative displacements from first to second position for

Table 1 Topologies with 1 common joint and 1-jointed branches, single branching.

Topology	Graph	Graph Solvability
$1-(1)$		Solvable, $m=3$
$1-(1,1)$		Solvable, $m=2.33$
$1-(1,1,1)$		Solvable, $m=2.14$
$1-(1,1,1,1)$		Solvable, $m=2.05$
$1-(1,1,1,1,1)$		Solvable, $m=2$

each finger j, $\hat{P}_j=\hat{P}_{2j}\hat{P}_{1j}^{-1}$, $j=1,\ldots,5$, where the composition of displacements is used.

The forward kinematics equations for relative displacements are constructed, for each finger, as the composition of screw displacements about each joint axes along the serial chain. The screw displacements about the joint axes are easily computed as the exponential of the unit twist, and so the product of exponentials is

$$e^{\hat{S}_0\frac{\Delta\theta_0}{2}}e^{\hat{S}_j\frac{\Delta\theta_j}{2}}, \quad j=1,\ldots,5, \tag{2}$$

where $\Delta\theta_j=\theta_j^2-\theta_j^1$ is the relative angle to transform from first to second position, for each finger j.

Impose now that each of the fingers has to be able to perform the desired relative displacement $\hat{P}_j$ from first to second position, to create a set of equations to solve for the design parameters of the robotic hand,

$$e^{\hat{S}_0\frac{\Delta\theta_0}{2}}e^{\hat{S}_j\frac{\Delta\theta_j}{2}}=\hat{P}_j, \quad j=1,\ldots,5. \tag{3}$$

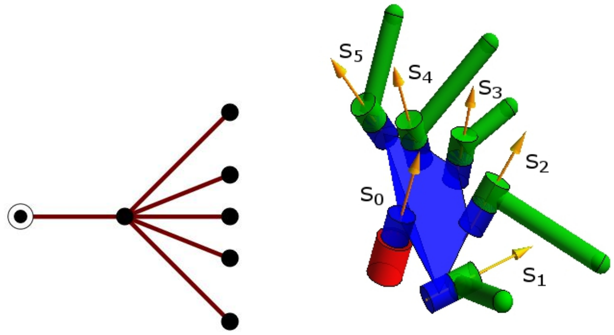


Fig. 1 The tree graph of the $1-(1,1,1,1,1)$ hand, left; the kinematic sketch, right.

This is a set of six independent equations per finger times five fingers, for a total of 30 independent equations. When using unit dual quaternions to express the displacements, the total set has 40 equations. The parameters to solve for are the Plucker coordinates of the axes and the relative joint angles to reach the relative transformation.

5.2 Algebraic solution

A revolute joint has five independent parameters, and is unable to perform a general displacement. Each finger of the $1-(1,1,1,1,1)$ hand is a serial $R-R$ chain in which the first joint cannot be fully arbitrary. This hand can be seen as five $R-R$ chains in which each chain needs to fully define the second joint axis and a single parameter of the common first joint axes, in order to perform a general displacement.

Using this rationale, solve for the second joint axes of each finger serial chain,

$$e^{\hat{S}_j \frac{\Delta\theta_j}{2}} = (e^{\hat{S}_0 \frac{\Delta\theta_0}{2}})^* \hat{P}_j, \quad j = 1,\dots,5, \tag{4}$$

where $*$ denotes the conjugate unit dual quaternion. The expansion of this equation in dual quaternion form yields:

$$\begin{aligned}&(\cos\frac{\Delta\theta_j}{2} + \varepsilon \sin\frac{\Delta\theta_j}{2})\hat{S}_j = \\ &(\cos\frac{\Delta\theta_0}{2} - \sin\frac{\Delta\theta_0}{2}\mathsf{S}_0)(p_{j0} + \varepsilon p_{j7} + \mathsf{P}_j), \quad j = 1,\dots,5,\end{aligned} \tag{5}$$

where,

$$\hat{S}_j = 0 + \varepsilon 0 + \mathsf{S}_j = 0 + s_{jx} i + s_{jy} j + s_{jz} k + \varepsilon(s^0_{jx} i + s^0_{jy} j + s^0_{jz} k + 0) \tag{6}$$

is the pure dual quaternion that represents a geometric line in the Clifford algebra. The solution for all $\mathsf{S}_j,\ j = 1,\ldots,5$ is obtained as a function of the first, common joint axis, S_0. These equations are also used to fully define this first axes.

Notice that the last component of the dual quaternion must be equal to zero for each joint solution, according to Eq. (5). This forces the product $(e^{\hat{S}_0 \frac{\Delta\theta_0}{2}})^* \hat{P}_j$ to have also last component equal to zero, and creates one equation on the parameters of the first axis S_0 from each finger equations,

$$\cos\frac{\Delta\theta_0}{2} p_{j7} + \sin\frac{\Delta\theta_0}{2}(\mathbf{s}_0 \cdot \mathbf{p}_j^0 + \mathbf{s}_0^0 \cdot \mathbf{p}_j) = 0, \quad j = 1,\ldots,5. \tag{7}$$

These are five equations in the five independent unknowns of the rotation about S_0 by $\Delta\theta_0$. Imposing that the joint angle has to be the same for the simultaneous task,

$$\cot\frac{\Delta\theta_0}{2} = \frac{-(\mathbf{s}_0 \cdot \mathbf{p}_j^0 + \mathbf{s}_0^0 \cdot \mathbf{p}_j)}{p_{j7}}, \quad j = 1,\ldots,5, \tag{8}$$

we end up with four linear equations in the Plucker coordinates of the axes. Those are six parameters subject to two Plucker constraints plus the four linear equations from (8),

$$\begin{aligned} &\frac{\mathbf{s}_0 \cdot \mathbf{p}_j^0 + \mathbf{s}_0^0 \cdot \mathbf{p}_j}{p_{j7}} = \frac{\mathbf{s}_0 \cdot \mathbf{p}_{j+1}^0 + \mathbf{s}_0^0 \cdot \mathbf{p}_{j+1}}{p_{(j+1),7}}, \quad j = 1,\ldots,4, \\ &\mathbf{s}_0 \cdot \mathbf{s}_0 = 1, \qquad \mathbf{s}_0 \cdot \mathbf{s}_0^0 = 0, \end{aligned} \tag{9}$$

with at most four solutions. However two of the solutions correspond to the double covering of $SO(3)$, and the final number of different solutions is two.

The two solutions for the Plucker coordinates of the first joint S_0 are used to compute a single rotation angle about this first joint for each solution, using Eq.(8). These values allow us to create the screw transformation about the first joint, and the rest of the joint axes and angles can be calculated using Eq.(4).

5.3 Example

Consider the simultaneous five-finger task composed of an initial position in which all fingers are spread and more or less evenly spaced in a circular space, to a final randomly-generated position. Figure 2 and Table 2 show the initial and final positions for this task.

The two solutions obtained are shown in Table 3 and Figure 3. The joint axes are expressed in the reference frame and when reaching the initial position. The design presented in Figure 4 is one of the infinitely many implementations that are possible for the kinematic solutions without changing the kinematic task.

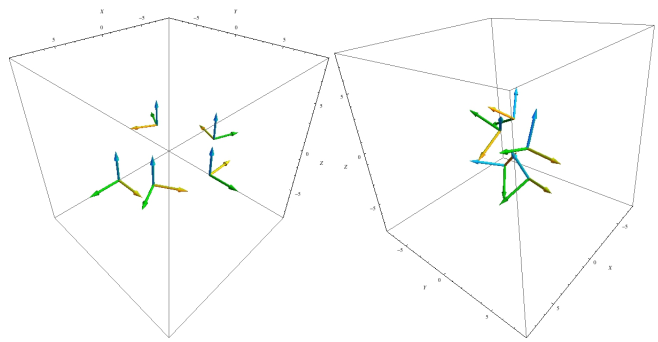


Fig. 2 Initial (left) and final (right) positions for the five fingers.

Table 2 Initial and final positions for each finger in dual quaternion form

Finger	Initial Position	Final Position
Finger 1	$1.0 + 3.0\varepsilon i$	$-0.96 - 0.11i - 0.19j + 0.16k + \varepsilon(-0.47i - 1.58j + 0.91k + 0.52)$
Finger 2	$0.97 + 0.26k + \varepsilon(2.41i + 0.65j)$	$-0.99 + 0.04i + 0.15j + 0.02k + \varepsilon(-0.63i - 1.62j - 0.38k - 0.29)$
Finger 3	$0.71 + 0.71k + \varepsilon(1.77i + 1.77j)$	$-0.62 - 0.38i + 0.08j + 0.68k + \varepsilon(0.37i + 0.75j - 0.53k - 0.70)$
Finger 4	$0.17 + 0.98k + \varepsilon(0.43i + 2.46j)$	$-0.68 + 0.39i - 0.005j + 0.62k + \varepsilon(-0.15i - 0.05j - 0.48k - 0.52)$
Finger 5	$0.50 - 0.87k + \varepsilon(1.25i - 2.16j)$	$0.81 + 0.41i + 0.42j + 0.07k + \varepsilon(0.63i + -0.33j - 0.75k - 0.08)$

6 Dimensional Synthesis for the $1-(1,1)$ Hand

Similarly, we can perform dimensional synthesis for the two-fingered to four-fingered hand topologies, according to the solvability results in Table 1. Specifying incomplete positions is not always a very practical approach. A better option could be to add some geometric constraints on the design so that this can be synthesized for $m = 2$ positions. Those geometric constraints could be pre-selecting one or more parameters on the joint axes, selecting the desired relative joint angles, or adding angle or distance constraints between consecutive axes. Depending on the nature of these additional constraints, the system of equations to solve will yield more ore less candidate designs. The example of the $1-(1,1)$ hand is developed here.

This hand has one joint at the wrist and a palm spanning two one-jointed fingers. Notice that this hand design is used in many simple hooks. This hand is solvable for $m = 2.33$ positions; if the designer imposes some constraints in the design parameters, the hand is solvable for exactly $m = 2$ positions. In particular, we need to define or constraint three of the unknown parameters.

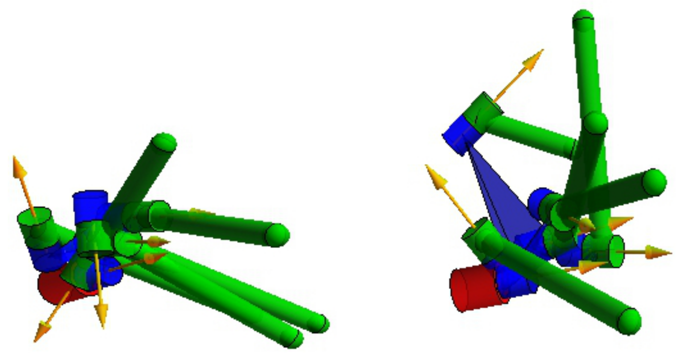


Fig. 3 The two solutions corresponding to the task in Figure 2.

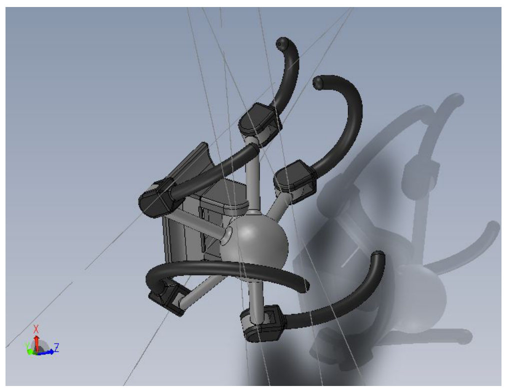


Fig. 4 The CAD model of the second solution.

For this example, the direction and relative angle of the wrist joint S_0 are pre-defined to values selected by the designer: $\mathbf{s}_0 = (0,0,1)$ and $\Delta\theta_0 = \pi/4$. This is one of the many possibilities in which the design can be partially pre-determined; in this case, a linear system with a single solution is obtained. The example uses the positions for fingers 1 and 2 from Table 2 to yield the design of Table 4 and Figure 5.

Similarly, two constraints can be imposed on the design parameters of the $1-(1,1,1)$ hand and one constraint on the $1-(1,1,1,1)$ hand in order to solve these for two exact positions. The solution process is similar to the one indicated for the five-fingered and one-fingered hands.

Table 3 Joint axes for the two solutions corresponding to the task in Figure 2.

Joint	Solution 1	Solution 2
Wrist (S_0)	$0.68i + 0.66j + 0.31k + \varepsilon(2.23i - 1.31j - 2.14k)$	$0.13i + 0.30j + 0.95k + \varepsilon(-1.41i + 1.53j - 0.29k)$
Finger 1 (S_1)	$0.50i + 0.80j + 0.31k\varepsilon(1.22i - 1.29j + 1.35k)$	$-0.18i + 0.22j + 0.96k + \varepsilon(-1.37i - 2.66j + 0.35k)$
Finger 2 (S_2)	$0.54i + 0.83j + 0.13k + \varepsilon(1.47i - 0.95j - 0.10k)$	$0.15i + 0.37j + 0.91k + \varepsilon(-0.76i - 1.62j + 0.79k)$
Finger 3 (S_3)	$0.62i + 0.73j - 0.28k + \varepsilon(1.63i + 2.18j + 2.04k)$	$-0.72i + 0.22j + 0.66k + \varepsilon(0.98i - 2.09j + 1.76k)$
Finger 4 (S_4)	$-0.81i + 0.11j - 0.58k + \varepsilon(-0.30i + 1.74j + 0.76k)$	$-0.68 + 0.39i - 0.005j + 0.62k + \varepsilon(7.33i - 7.33j + 3.36k)$
Finger 5 (S_5)	$-0.96i + 0.14j + 0.23k + \varepsilon(0.28i + 4.30j - 1.48k)$	$0.12i - 0.79j + 0.60k + \varepsilon(-1.18i + 0.68j + 1.14k)$

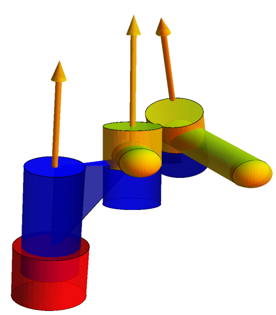


Fig. 5 The two-fingered design for the first two finger tasks in Figure 2

Table 4 Joint axes for the $1-(1,1)$ hand designed for the first two fingers of the task in Figure 2.

Joint	Solution
Wrist (S_0)	$1k + \varepsilon(3.46i + 2.34j)$
Finger 1 (S_1)	$-0.31i - 0.24j + 0.92k + \varepsilon(4.57i - 3.13j + 0.70k)$
Finger 2 (S_2)	$0.10i + 0.23j + 0.97k + \varepsilon(4.08i - 1.07j - 0.15k)$

7 Conclusions

Kinematic synthesis can be used to yield multi-fingered hand designs in which the task of each finger is simultaneously defined. A big variety of hand topologies and tasks can be used to yield hand designs. In this work, we present the solution of one of the simplest family of hands, that of a wristed hand with a single revolute joint at the wrist and a number of fingers, each of them attached to the palm by a single revolute joint. It can be proved that we can design hands with two to five fingers able to perform a pick-and-place task where each finger's position can be exactly specified.

For these four hand topologies, a solution method is developed and shown to yield two hand solutions in the general case. Two examples are presented, one for the five-fingered hand and another one for the two-fingered hand.

Task-oriented hand design may lead to more optimal end effectors. The task specification can be complemented with the definition of velocities and accelerations for grasping characterization. Those constraints can be added to the design process without increasing the complexity of the system of equations.

Acknowledgements This work is supported by the National Science Foundation under Grant No. 1208385. The content is solely the author's responsibility.

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